**Chapter 9 – Linear Transformer**

**P9.1.12** Two coils having *N*1 = 800 turns and *N*2 = 500 turns are coupled through a high-permeability core. A current  in coil 1 results in *φ*11eff(leak) = 500 μWb and *φ*21 = 400 μWb, whereas a current 2*i*1 in coil 2 results in *φ*22eff(leak) = 1400 μWb. Determine: (a) *φ*12 resulting from 2*i*1 in coil 2; (b) the coefficient of coupling; (c) the mutual inductance, assuming that the permeance of the core is 50 nWb/A-turn; (d) the inductance of each coil.

**Solution:** (a) Since *M*12 = *M*21: , so *φ*12 = 500 μWb.

1. From Equation 9.2.14, *k* =  = 0.342.
2. *M* = *N*1*N*2*P*c = 800×500×50×10-9 = 20 mH.
3. *L*1 = , *L*2 = ; hence, = 1.516, and *L*1*L*2 = . This gives *L*1 = 72 mH and *L*2 = 47.50 mH.



**P9.1.15** Determine *Leq* in Figure P9.1.15.

**Solution:** If a current **I** flows in the inductors, the voltage induced in the 3 H inductor is *jω***I**(3 – 1 + 1) = *jω*3**I**; the voltage induced in the 2 H inductor is *jω***I**(2 – 1 – 1) = 0; the voltage induced in the 1 H inductor is *jω***I**(1 – 1 + 1) = *jω***I**; **V** = *jω*3**I** + *jω***I** = *jω*4**I**. It follows that *Leq* = 4 H.



**P9.1.18** Given the coupled coils of Figure P9.1.18, with *iSRC* being the triangular waveform shown. Sketch the waveforms of *v*1 and *v*2.



**Solution:** *M* = 0.8 mH. The voltage across the primary winding is .  is (4 A)/(2 ms), or 2000 A/s;*v*1 is a square waveform having an amplitude of 45×10–3×2000 = 90 V.



The open-circuit secondary voltage is ; *v*2 is thus a square wave of amplitude

24×10–3×2000 = 48 V but of phase opposite to *v*1.

The ratio of the amplitudes of *v*2 to *v*1 is .



**P9.2.7** Determine *M* in Figure P9.2.7 so that *i* = 0.

**Solution:** The circuit in the frequency domain is as shown. When *i* = 0, **Vbc** = 0, which means , or . This gives 3(2 – *M*) = 4 – *M*, or *M* = 1 H.



**P9.2.12**Determine the frequency at which the current  in Figure P9.2.12 has the same magnitude when the connections of one coil are reversed.



**Solution:** *M* = 0.75 = 45 mH. Since the current magnitude remains the same when one coil is reversed, the impedance reverses sign. Thus: *jω*(30 + 120 + 90) –  = -*jω*(30 + 120 - 90) + , or 300*ω* = . *ω*2 = ; *ω* =  krad/s.



**P9.2.15** Determine the stored energy in the circuit of Figure P9.2.15 in the dc steady state, assuming *M* = 1 H.

**Solution:** No current flows in the dc state through the 4 H inductor because of the series capacitor. *Leq* of the 2 H and 3 H inductors is 2 + 3 + 2×1 = 7 H. This inductance acts as a short circuit, so that the current through it is 1/0.5 = 2 A, and the voltageof the capacitor is zero. The energy stored in the circuit is (½)×7×4 = 14 J.



**P9.2.22** Derive TEC looking



into terminal ‘ab’ in

Figure

P9.2.22.

**Solution:** Replacing the linear transformer by its T-equivalent circuit,



it is seen that on open circuit, the

-*j*40 Ω in series with *j*10 Ω gives -*j*30 Ω. In parallel with *j*30 Ω, this results in an open circuit. The current through the *j*5 Ω is zero, so that both terminals of this inductor are at a voltage 5**IX**. The current in the -*j*5 Ω impedance is **ISRC** – **Ix**. From KVL, 10**Ix** + *j*5(**ISRC** – **Ix**) = 5**Ix**. This gives  A, and 5**IX** = 20 V. It follows that **VTh** = **Vab** =  = 26.67 V.

On short circuit, the voltage of the dependent source is still 20 V. The parallel impedance of *j*10 Ω and *j*30 Ω is *j*7.5 Ω. The current through the *j*5 Ω impedance is 20/(*j*12.5) A, From current division, **ISC** is this current multiplied by 3/4. This gives **ISC** = -*j*1.2 A. It follows that *ZTh* = (800/3)/(-*j*1.2) = *j*200/9 = *j*22.22 Ω.



**P9.2.23** Derive TEC between terminals ‘ab’ in Figure P9.2.23.



**Solution:** With terminals ab are open circuited, the equivalent imprdance in mesh 1 is *j*50 + *j*40 + 2×*j*30 = *j*150 Ω. Hence, **I1** =   A. **VTh** = (*j*40 – *j*100 + *j*40 + *j*30 + *j*20)**I1** = *j*60(1 – *j*) = 60(1 + j) V.



With terminals ab short circuited, 200 = (50 + *j*50 + *j*40 – *j*100)**I1** – (*j*40 – *j*100)**I2** + *j*30**I1** + *j*30(**I1** – **I2**) – *j*40**I2** – *j*20**I2**, or (50 + *j*50)**I1** – *j*30**I2** = 200. For mesh 2: (*j*80 + *j*40 – *j*100)**I2** – (*j*40 – *j*100)**I1** + *j*20**I2** +*j*20(**I2** – **I1**) – *j*30**I1** – *j*40**I1** = 0, or –*j*30**I1** + *j*60**I2** = 0, or **I1** = 2**I2**. Substituting, **Isc** = **I2** A. *ZTh* =  Ω.



**P9.2.31** Determine the mesh currents in Figure P9.2.31.

**Solution:** Mesh 1:

(*j*30 + *j*10)**I1** – *j*10**I2** – (*j*30 – *j*10)**I3** =

20∠50°

Mesh 2: -*j*10**I1** + (50 + *j*10 – *j*30)**I2**

-(50 + *j*10)**I3** = 0

Mesh 3: -(*j*30 – *j*10)**I1** – (50 + *j*10)**I2** + (50 + *j*30 + *j*20)**I3** = 0